Hydrodynamic theory of quantum fluctuating superconductivity

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Hydrodynamic description of conventional metals

- **Hydrodynamics:**
  - Universal low energy, long wavelength physics.
  - Conserved charges, their currents, Goldstone bosons.

- **Conservation law:**
  \[ \frac{\partial \rho}{\partial t} + \nabla \cdot j = 0 \]

- **Constitutive relation** (derivative expansion):
  \[ j = -D \nabla \rho + \cdots \]

- **Conductivity** (Einstein relation):
  \[ j = -D \nabla \rho = -D \frac{\partial \rho}{\partial \mu} \nabla \mu = D \chi E = \sigma E \]
Comment on screening by Maxwell fields

• Charge in a metal does not diffuse, it decays exponentially.

• This comes from solving Maxwell’s equations + Ohm’s law.

• The Einstein relation for the conductivity still holds.

• $\sigma$ measured with respect to total, not external, electric field:

$$\mathbf{j} = \sigma \mathbf{E}_{\text{tot}} = \sigma \frac{E_{\text{ext}}}{\varepsilon(\omega, k)} = \frac{\sigma E_{\text{ext}}}{1 - \frac{1}{k^2} \chi(\omega, k)} = \frac{-i\omega D\chi}{i\omega - D(k^2 + \chi)} E_{\text{ext}}$$
Superfluid hydrodynamics

- Phase $\phi$ of the order parameter appears in hydrodynamics.

- $u_\phi = \frac{1}{m} \nabla \phi$ is the superfluid velocity.

- ‘Josephson relation’:
  \[
  m \frac{\partial u_\phi}{\partial t} = \nabla \frac{\partial \phi}{\partial t} = -\nabla \mu + \cdots
  \]

- Constitutive relation:
  \[
  j = \frac{\rho_s}{m} u_\phi - D \nabla \rho + \cdots
  \]

- (super-)Conductivity:
  \[
  j = - \left( \frac{\rho_s}{m^2} \frac{i}{\omega} + D \chi \right) \nabla \mu = \left( \frac{\rho_s}{m^2} \frac{i}{\omega} + D \chi \right) E = \sigma(\omega) E
  \]
Superconductivity

• $\propto$ conductivity because: diffusion $\rightarrow$ second sound mode.

• In a superconductor, the U(1) symmetry is gauged, i.e. coupled to electromagnetism.

• This gaps out the Goldstone/sound mode in the same way the diffusive mode was previously gapped.

• However, the conductivity is, as before, measured with respect to the total electric field. So the unscreened (superfluid) hydrodynamics determines the conductivities.
Vortices and supercurrent relaxation

- In two space dimensions, above picture incomplete.

- Motion of vortices can wind and unwind the supercurrent.

\[ \Delta \phi = 2\pi \]

- Expect supercurrent relaxation rate \( \Omega \):

\[
\sigma(\omega) = \frac{\rho_s}{m^2} \frac{1}{-i\omega + \Omega}
\]
Vortices and supercurrent relaxation

- This problem is well understood in some regimes:
  → Thermal BKT proliferation of vortices above $T_{\text{BKT}}$.

- Classical picture: vortices pushed across sample by ‘superfluid Magnus force’
  → The core of the vortices is in the normal state.
  → Therefore, motion of vortices creates dissipation.
  → Get

\[ \Omega \sim \frac{n_f A_v}{\sigma_n} \quad \text{[Bardeen-Stephen ’65]} \]

- Much controversy, however, about whether (quantum) phase-disordered superconductors exist at $T = 0$. 
In the remainder

• Lightening overview of some experiments.

• Develop a **fully quantum effective field theoretic formalism** for the conductivity of phase-disordered superconductors.

• Illustrate formalism with two examples:

  (i) ‘Check’: Elegant (re)derivation of Bardeen-Stephen result.

  (ii) Phase disordering by a Chern-Simons interaction [‘topologically ordered superfluid vortex liquid’].
Superfluid-insulator transitions

• In two (spatial) dimensions, conventional theory suggests that as $T \to 0$ electrons will either localize or pair up.

• That is, the phase of matter one expects to find is either an insulator or a superconductor.

• Indeed, early experiments suggested that disordered thin films undergo superconductor-insulator transitions as a function of magnetic field or thickness ($\approx 1/$disorder).

Destroys superconductivity

Favors localization
Superfluid-insulator transitions

[Hebard and Paalanen '90, α-InO$_x$]

[Jaeger et al. '89, Pb]
Metallic phases in two dimensions

- Problematically for ‘conventional’ understanding, in weakly disordered films a metallic phase intervenes (at $T = 0!$) between the superconductor and insulator.

[Mason, Kapitulnik ’99, α-MoGe]
Metallic phases in two dimensions

- Often, the residual resistivity of the metallic phase is much smaller than the “normal state” resistivity of the material at temperatures above the “mean field” superconducting temperature.

- Suggests the low energy degrees of freedom of the metallic phases are not the normal state quasiparticles.

- Natural to think of as “failed superconductors” where (quantum!) phase fluctuations have destroyed phase coherence.
Metallic phases in two dimensions

- Direct motivation for our work: observation of a Drude-like peak in the metallic phase of InO$_x$.

[Liu, Pan, Wen, Kim, Sambandamurthy, Armitage ’13]
Metallic phases in two dimensions

- The width of the Drude-like peak goes to zero at the same magnetic field where superconductivity appears.

[Liu, Pan, Wen, Kim, Sambandamurthy, Armitage ’13]
Memory matrix formalism

• Most discussions of this physics have involved semi-microscopic models with uncontrolled approximations.

• Instead: work in a limit where a hierarchy of timescales allows an effective field theoretic approach.

• Small parameter will be the supercurrent relaxation rate. I.e. want $\Omega \ll T$, etc.

• (Approach inspired by studies in holographic systems over past few years, where slow mode was momentum.)
Memory matrix formalism

• Suppose that $H = H_0 + \varepsilon \Delta H$, with $[\Delta H, J_\phi] \neq 0$.

• Then the decay of $J_\phi$ is slow and dominates $\sigma$:

$$\sigma(\omega) = \frac{\chi^2 \rho \phi}{\chi J_\phi J_\phi} \frac{1}{-i\omega + \Omega} + \cdots$$

• But now we have a formula for $\Omega!$:

$$\Omega = \varepsilon^2 \frac{1}{\chi J_\phi J_\phi} \lim_{\omega \to 0} \text{Im} \left( \frac{G^R_{i[\Delta H, J_\phi]} i[\Delta H, J_\phi]}{\omega} \right)_{\varepsilon=0}$$

Spectral density of states into which $J_\phi$ can decay. Cf. Fermi Golden rule.
Supercurrent relaxation

• Recap: if an ‘almost conserved’ operator carries current, rate of the decay determines the conductivity.

• In our case of interest today: \( J_\phi = \frac{1}{m} \int d^2 x \nabla \phi \)

• Need an interaction that doesn’t commute with \( J_\phi \).

• Natural building block:
  \[
  \pi_\phi = \frac{\partial f}{\partial \phi} = -\frac{\partial f}{\partial \mu} = \rho.
  \]
  i.e. charge density is canonically conjugate to the phase:
  \[
  [\phi(x), \rho(y)] = i\delta(x - y).\]
Supercurrent relaxation

• Thus a simple, generic perturbation of the superfluid state is the short range Coulombic interaction:

\[ \Delta H = \frac{\lambda}{2} \int d^2 x \rho(x)^2. \]

• At first glance looks like commutator is trivial total derivative:

\[ i[\Delta H, J_\phi] = -\frac{\lambda}{m} \int d^2 x \nabla \rho(x) \]

• However, the phase appearing in \( J_\phi \) is only defined outside of vortex cores! Above integral is then also only over the outside of vortex cores. Integral over all space vanishes: \( \rightarrow \) integral over vortex cores.
Supercurrent relaxation

- The memory matrix formula for $\Omega$ becomes an integral of the two point function of $\rho$ over the vortex core.

- Using the diffusive behavior of $\rho$ in normal state, the Bardeen-Stephen formula drops out exactly.

  $$\Omega \sim \frac{n_f A_v}{\sigma_n}$$

- So we discover the quantum origin of this formula. Charge interactions enhance phase fluctuations:

  $$\Delta \rho \Delta \phi \gtrsim \hbar$$
Supercurrent relaxation without parity

- With parity and time-reversal broken, a second very natural $\Delta H$ exists.

- Suppose the low energy effective theory is coupled to an emergent Chern-Simons gauge field:

  $$L = L_{\text{matter}} + j_\mu (A^\mu + a^\mu) - \frac{1}{2\lambda'} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

- Integrating out the gauge field generates

  $$L' = \frac{\lambda'}{2} j_\mu \frac{\epsilon^{\mu\nu\rho}}{\partial_\sigma \partial_\sigma} j_\nu \quad \Rightarrow \quad \Delta H = \frac{\lambda'}{2} \int \frac{d^2 k}{(2\pi)^2} \frac{\rho_{-k} (\nabla \times j)^{\hat{z}}_k}{k^2} + \text{h.c.}$$
Supercurrent relaxation without parity

- **Non-locality** of induced interaction leads to a nonzero time dependence of $J_\Phi$ everywhere. In fact:

$$i[\Delta H, J_\phi^i] = -\frac{\chi'}{m} \epsilon^{ij} J_j^j.$$

- Rough physical picture:
  Current = Flow of charge
  → Flow of emergent magnetic flux (CS term)
  → Flow of vortices
  → Relaxation of supercurrent in transverse direction!

- $\Omega$ depends on charge flow in normal component.
Supercurrent relaxation without parity

- Result for conductivities:

\[
\sigma_{xx} = -\frac{m^2}{\lambda'^2 \rho_s} \frac{\omega (\omega \Omega + i(\Omega^2 + \Omega_H^2))}{(-i\omega + \Omega)^2 + \Omega_H^2},
\]

\[
\sigma_{xy} = -\frac{1}{\lambda'} - \frac{m^2}{\lambda'^2 \rho_s} \frac{\omega^2 \Omega_H}{(-i\omega + \Omega)^2 + \Omega_H^2},
\]

- Feature: ‘supercyclotron resonance’ at

\[
\omega_* = \pm \Omega^H - i\Omega = \frac{\lambda' \rho_s}{m^2} \frac{1}{\pm 1 - \lambda' (\pm \sigma_0^H - i\sigma_0)}.
\]

Conductivities of the normal component of superfluid.
Superfluid relaxation occurs if perturbations of effective Hamiltonian do not commute with the supercurrent.

Starting with perturbations of superfluid hydrodynamics gives controlled entry point. This works even if the underlying microscopic dynamics is strongly correlated.

Gave two examples, with and without parity:
(2) Without parity: ‘supercyclotron resonance’ determined by conductivities of normal component.