Cosmological Evolution of Gamma-Ray Bursts

Ellie Kitanidis
June 2013

AN UNDERGRADUATE HONORS THESIS
SUBMITTED TO THE DEPARTMENT OF PHYSICS
AND THE UNDERGRADUATE STUDIES COMMITTEE
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
BACHELOR OF SCIENCE WITH HONORS
I certify that I have read this dissertation and that in my opinion it is fully adequate, in scope and quality, as an Undergraduate Honors Thesis.

__________________________________
Vahé Petrosian, Principal Advisor

I certify that I have read this dissertation and that in my opinion it is fully adequate, in scope and quality, as an Undergraduate Honors Thesis.

__________________________________
Sarah Church

Approved by the Physics Department

__________________________________
Table of Contents

1. Introduction
   1.1 History and Overview
   1.2 GRBs as Cosmological Tools
2. Procedure
   2.1 Lynden-Bell C: Method
   2.2 Efron & Petrosian 1992 Method
3. Testing with Simulated Data
4. Swift Data
   4.1 Flux Cut
   4.2 K-Correction
5. Results
   5.1 Luminosity Evolution
   5.2 Luminosity Function
   5.3 Co-Moving Density Rate Evolution
   5.4 Comparison to Star Formation Rate
   5.5 logN-logS and Comparison with BATSE
6. Discussion
7. Summary
8. References
1 Introduction

1.1 History and Overview

Gamma-ray bursts were discovered serendipitously. In the early 1960s, the U.S. government launched the Vela satellites, designed to monitor for signs which might suggest the USSR was violating the Nuclear Test Ban Treaty by conducting nuclear weapons tests in space. Yet on July 2nd, 1967, the Vela satellites detected a burst of gamma radiation that did not seem to match the signature of a nuclear detonation. In fact, it didn’t match the signature of anything the team at Los Alamos had ever seen before. It soon became clear that they were dealing with an entirely new celestial object.

Many theories flourished in the early days of GRB research, and the widely accepted model throughout the 1980’s posited relatively nearby neutron stars as sources. This all changed in 1991, when NASA launched the Compton Gamma Ray Observatory, carrying with it the Burst and Transient Source Experiment (BATSE). BATSE consisted of a large array of detectors which could detect gamma-rays in the 50keV-1MeV range from all directions, regardless of the alignment of the spacecraft. The results from these observations showed that the GRBs were isotropically distributed, which, along with a noticeable paucity of weak bursts, seemed to hint at their cosmological origin. This theory was confirmed when the first redshift measurements of host galaxies were obtained from BeppoSAX, an Italian-Dutch satellite which used X-ray localizations to identify host galaxies.
Despite over 40 years of study, many aspects of GRBs remain highly mysterious, though promising theories have emerged regarding their origins and emission mechanisms. Bursts seem to belong to one of two distinct classes, categorized as either long (duration > 2 seconds) or short, and are thought to have different progenitors. Long GRBs, the only type we will concern ourselves with in this paper, are believed to be associated with the deaths of massive stars whose iron cores collapse to black holes or neutron stars. The theory describing how long gamma-ray bursts are produced from these supernovae is called the “collapsar” model (MacFadyen & Woosley 1999; Woosley & Bloom 2006). The emission mechanism can be summarized as follows: when the core collapses, the outer layers of the star explode away from it at relativistic speeds. Inhomogeneity in the outflow produces shells of material with different Lorentz factors, which collide in a series of internal shocks and release radiation through synchrotron processes. The ejected material is then believed to coalesce into a single shell, with an external shock traveling into the external medium and heating it up, producing more photons at lower energies such as X-ray, optical, and radio. This creates the “afterglows” that accompany many long gamma-ray bursts. GRBs which lack afterglows, called “dark bursts,” are thought to be too distant for the lower frequency components to be detected.
1.2 GRBs as Cosmological Tools

The ever-growing supply of data which include GRBs with very high redshift (up to \( z \sim 9 \)), made available by instruments on board BeppoSAX, HETE, INTEGRAL and particularly Swift, has inspired a great deal of interest in using them as cosmological tools - either to constrain cosmological parameters, such as equations of state and density parameters, or as probes of the early universe.

There have been many attempts to do the former by searching for distance-dependent characteristics which are “standard candles” or correlated with distance-independent characteristics (Norris et al. 2000; Reichart et al. 2001; Amati et al. 2002; Lamb et al. 2004; Attiea et al. 2004; Ghirlanda et al. 2004). For example, the Amati relation proposes a correlation between the peak energy \( E_P \) of the \( \nu F_\nu \) spectrum and the total isotropic energy \( \epsilon_{iso} \), \( E_P \propto \epsilon_{iso}^{0.6} \). Similarly, Ghirlanda relates \( E_P \) to the beaming corrected energy \( \epsilon_\gamma \), \( E_P \propto \epsilon_\gamma^{0.7} \).

However, uncertainties in these supposed relations, analyzed in detail by Petrosian et al. (2009), suggest that they are unreliable, as the characteristics have broader distributions and more complex correlations than assumed (see also Lloyd et al. 2000, Nakar & Piran 2004, Band & Preece 2005, Butler et al. 2009).

Furthermore, \( \epsilon_{iso} = 4\pi d_L^2 F_{tot}/(1+z) \), where \( F_{tot} \) is the total fluence (flux integrated over time and energy), and the luminosity distance \( d_L \) is a function of redshift and the cosmological parameters. Thus, even assuming a simple
one-to-one correlation, such as, for example, \( \epsilon_{iso} = \epsilon_0(z)f[z,E_p/E_0(z)] \), the cosmological parameters (specifically, the density parameter \( \Omega(z) \) and the Hubble constant \( H_0 \)) are intrinsically linked to observation as

\[
4\pi(1+z)
\left(
\frac{c}{H_0}
\right)^2
\left[
\int_0^z \left[\Omega(z')\right]^{-1/2} dz'
\right]^2
= \epsilon_0 f[z,E_p^{obs}(1+z)/E_0(z)]/F_{tot}
\]

Thus, as Petrosian concludes, it is impossible to obtain both the cosmological parameters and the cosmological evolution of the various functions (e.g. \( \epsilon_0(z), E_0(z) \)) in this way; a form for one must be assumed in order to determine the other, or else it descends into circular reasoning. For now, it seems, the use of GRBs to constrain cosmological parameters is not a fruitful approach.

Fortunately, using them as cosmological probes appears to be a much more promising direction. For this purpose, one assumes the values of the cosmological parameters and attempts to determine the correlations and distributions of the relevant functions, which in this case are the redshift and \( \epsilon_{iso} \sim \) peak luminosity \( L = 4\pi d_L^2(z,\Omega_i)f \) functions from the measured flux \( f \) and redshift \( z \). In what follows, we assume a flat universe with \( H_0 = 71 \text{ km/Mpc/s}, \Omega_m = 0.27 \) for matter, and \( \Omega_\Lambda = 0.73 \) for the cosmological constant. Using this model, we can obtain the luminosity distance,

\[
d_L(z) = (1+z)
\frac{c}{H_0}
\int_0^z
\sqrt{
\frac{1}{\Omega_m(1+z')^3 + \Omega_\Lambda}
} dz'
\]
Lately, there has been interest in using the GRB rate as a tracer for star formation rate (SFR) or cosmic metallicity evolution (CME). Consequently, there has been increased activity in the investigation of the luminosity function and evolution and the co-moving formation rate density of GRBs in relation to star formation (see e.g. Porciani & Madau 2001; Natarajan et al. 2005; Daigne et al. 2006; Jakobson et al. 2006; Salvaterra et al. 2009). However, these methods inevitably involve many assumptions, some of which are not substantiated by GRB or other observations. For example, it is common to assume a parametric form for the GRB formation rate (GRBFR) based on the SFR, while a more reliable procedure would be to determine the form of the GRBFR directly from the data using the non-parametric methods developed by Efron & Petrosian (1992, 1994, 1999). This type of analysis has already been proven successful in studying other characteristics of GRBs (see Lee & Petrosian 1996, 1997; Lloyd et al. 2000 and 2002; Kocevski & Liang 2006). One effect in particular that must be accounted for is the Malmquist bias or flux-limited sampling (see Malmquist 1920; Eddington 1915,1940; Neymann & Scott 1959), in which the limited sensitivity of the detector causes truncation of the data. In our method, we use non-parametric procedures to correct for this effect. To see a more detailed review of these techniques and a history of the Malmquist bias, see Petrosian (1992, 2002).
2 Procedure

2.1 Lynden Bell $C^-$ Method

We are primarily interested in the co-moving density rate evolution $\dot{\rho}(z)$. Since this is obtained from a bivariate luminosity-redshift data set, its determination also involves the determination of the luminosity function $\psi(L)$ and its evolution. The methods that we use here give GRBFR directly, or more precisely, the cumulative distributions $\dot{\sigma}(z)$ and $\phi(L)$:

$$\dot{\sigma}(z) = \int_0^z \frac{\dot{\rho}(z')}{1+z'} \frac{dV}{dz'} dz'$$

$$\phi(L) = \int_L^\infty \psi(L') dL'$$

However, as mentioned previously, the data is truncated due to the Malmquist bias. Thus, simply counting the objects does not take into account the flux cutoff imposed by the detector, which limits the sample fluxes ($f > f_{\text{min}}$, or $L > 4\pi d_L^2(z, \Omega_i) f_{\text{min}}$). This selection effect must be corrected for in order to determine the unbiased distributions $\psi(L)$ and $\dot{\rho}(z)$.

If the luminosity and redshift are uncorrelated (i.e. there is no luminosity evolution), then there are several methods to obtain these functions. However, as shown in Petrosian (1992), the best method is the maximum likelihood technique first applied by Lynden Bell in the context of flux-limited quasar studies. The Lynden Bell $C^-$ method, as it is known, estimates underlying parent distributions non-parametrically. Once the redshift and luminosity distributions have been obtained, they can be used to determine
the above cumulative distributions. In addition to requiring that the luminosity and redshift are uncorrelated, the Lynden Bell $C^{-}$ method necessitates a well defined truncation limit. This latter condition is seemingly satisfied in our case, since our data set is relatively large, and all acquired from the same detector, thus ensuring the threshold is consistent between bursts.

Figure 1: The basic principle of the Lynden-Bell $C^{-}$ method. Here, we show the $L$-$z$ distribution of a sample of Swift bursts, with the blue line representing the flux cutoff induced by the detector threshold. This truncation can be corrected for using the Lynden Bell $C^{-}$ method, which estimates underlying parent distributions non-parametrically. The key idea is that if $L$ and $z$ are uncorrelated, we expect that the true number of objects in some bin $X$, which might cross the truncation boundary, can be related to the number of objects in other bins that are not truncated, by a series of ratios.

The essence of the Lynden Bell method is illustrated in Figure 1. We use a technique that is essentially equivalent to the $C^{-}$ method, outlined below:
For some burst $i$, consider the box defined by $L_i < L < \text{and } 0 < z < z_{\text{max}}(L_i)$, where $z_{\text{max}}(L_i)$ is the maximum redshift that can be observed at that luminosity $L_i$ without crossing the truncation boundary. The number of events contained within this box is called the “associated set” $N_i$. Similarly, the number of bursts in a box defined by $0 < z < z_i$ and $L_{\text{min}}(z_i) < L < \infty$, where $L_{\text{min}}(z_i)$ is the minimum observable luminosity corresponding to the redshift $z_i$ within the region bounded by the truncation, is called the “associated set” $M_i$. In other words, for each $(L, z)$ point, we can construct two data sets, each defined by either a minimum luminosity or a maximum redshift that can be observed. An illustration of the boundaries of $N_i$ and $M_i$ are shown in Figure 2. We use the $N_i$ associated sets with the Lynden Bell $C^-$ method to determine the cumulative distribution for the y-axis, while the $M_i$ associated sets give us the cumulative distribution for the x-axis.

Now, let the differential $dN_i$ be the number of points between $L_i$ and $L_i + dL_i$. Assuming the luminosity and redshift are stochastically independent, we should expect that

$$\frac{dN_i}{N_i} = \frac{d\phi}{\phi_i}$$

which can then be integrated (or rather, summed, for discrete data) to find $\phi(L)$:

$$\phi(L_i) = \sum_{k=2}^{i} (1 + \frac{1}{N_k})$$

Note that we do not include the $L_i$ object that is used to form the associated set in our count. Also note that both the cumulative distributions
and their corresponding differential functions are technically undefined in the case of empty associated sets.

In much the same way, we can determine $\dot{\sigma}(z)$ by using the associated set $M_i$, which represents the number of bursts in the box $0 < z < z_i$ and $L_{\text{min}}(z_i) < L < \infty$. We have:

$$\dot{\sigma}(z_i) = \sum_{k=2}^{i} \left(1 + \frac{1}{M_k}\right)$$

The normalization for $\phi(L)$ and $\dot{\sigma}(z)$ is arbitrary, so only the shape of the distributions can be determined, not their absolute densities. The normalization can later be determined by fitting the expected source count to
the observed source count.

\section{2.2 Efron & Petrosian 1992 Method}

For the Lynden-Bell \( C^- \) method to be applicable, the luminosity and redshift must be stochastically independent. Thus, it is necessary to determine the luminosity evolution and remove it from the data in order to proceed with our analysis. Suppose we have a bivariate data set \( \{x_i, y_i\} \) of \( N \) points to be tested for independence. If \( x \) and \( y \) are stochastically independent, we expect the rank \( R_i \) of \( x_i \) to be uniformly distributed between 1 and \( N \), thus having expectation \( E = \frac{1}{2}(N + 1) \) and variance \( V = \frac{1}{12}(N^2 - 1) \). In fact, we can construct a specialized version of the Kendall \( \tau \) test statistic which normalizes the rank \( R_i \) such that \( \tau = 0 \) indicates independence and \( \tau = 1 \) indicates a 1\( \sigma \) correlation. This normalized rank statistic is defined as

\[ \tau = \frac{\sum_i (R_i - E)}{\sum_i V} \]

However, detector selection effects complicate this procedure, since any attempt to directly calculate the correlation without accounting for truncation results in a dramatic overestimation of its strength. The effect of the data truncation is to augment the apparent correlation between the two variables, so we must use a method which takes this effect into account.

The Efron & Petrosian 1992 method uses a modified version of the Kendall \( \tau \) statistic test the independence of truncated data. Rather than using the
ranks in the entire data set, we once again make use of the concept of the “associated sets” \( N_i \) and \( M_i \), introduced in Section 2.1, which are unaffected by the flux limit. By using the ranks in the associated sets, we avoid the false correlation due to the truncation.

Now we expect the ranks to be uniformly distributed between 1 and \( N_i \), with expectation \( \frac{1}{2}(N_i + 1) \) and variance \( V = \frac{1}{12}(N_i^2 - 1) \). The new Kendall \( \tau \) rank correlation coefficient is given by

\[
\tau = \frac{\sum_i (R_i - E_i)}{\sum_i V_i}
\]

Here, \( E_i \) and \( V_i \) refer to the mean and variance, respectively, of the set of sorted \( y \)'s which are allowed to pair with \( x_i \) (the associated set). \( R_i \) refers to the rank of the \( y \), within that subset, which corresponds to \( x_i \) in the actual observed data. These values are summed over all associated sets to produce \( \tau \), which represents the degree of correlation with proper accounting for truncation. A value of 0 indicates independence of the variables. Any correlation between \( L \) and \( z \) (i.e., \( \tau > 1 \)) implies the presence of luminosity evolution at a one sigma level.

We can determine the luminosity evolution by first choosing a functional form for it, \( g(z) \). We can then make the variable transformation by defining a new luminosity \( L_0 = L/g(z) \) and vary the parameters of the function until \( \tau \) approaches 0, indicating that \( L_0 \) and \( z \) are independent. Once independence is established, we can use the Lynden-Bell \( C^- \) method to determine the
This method, as well as the Lynden Bell $C^-$ Method, have the advantage of being equipped to deal with the most general truncation case, where each data point has different limits in $L$ and/or $z$. Hence, we can use this method even when our data comes from multiple sources with different detector thresholds.

3 Testing with Simulated Data

We first test our method on a very large simulated data set (38,224 bursts) with known luminosity function $\psi(L)$ and density rate $\dot{\rho}(z)$. We assume that the data has already been corrected for luminosity evolution, and apply a flux cut which reduces the population to 9,501 bursts, as shown in Figure 3.

In Figure 4 and Figure 5, we show the corrected cumulative luminosity and redshift distributions, as well as the “raw” distributions created without accounting for truncation effects, i.e. by simply counting up objects normally. For comparison, we also show the true distributions which were assumed.

As expected, the Lynden-Bell method reproduces the true distributions very accurately, except at low redshifts and luminosities where the number of objects in the associated sets $(N_i,M_i)$ are small and the error bars are large. The uncorrected distributions diverge, since naively counting up objects leaves us blind to the population below the detection threshold. Specifically, the method which doesn’t account for truncation misses more objects
at low luminosities and high redshifts.

When the flux limit is set to zero (in other words, for untruncated data), the two methods yield matching results, with Kolmogorov-Smirnov probability of 1, as expected.

4 Swift Data

We next apply our method to a set of Swift bursts which give peak photon flux in the 15-150keV band. NASA’s Swift is a multi-wavelength space observatory, named in reference to its rapid localization of bursts. Aside from the burst alert telescope, it also has instruments designed to perform spec-
Figure 4: Comparison of uncorrected, corrected, and “true” luminosity cumulative distributions.

tral analysis of the X-ray and UV/optical afterglows. This makes it ideal for redshift determination (since the prompt emission spectra lack well defined features and are ill-suited to spectral analysis), leading to an unprecedented number of bursts with known redshift. We can later add non-Swift bursts, including those detected by telescopes such as BATSE, HETE, and INTEGRAL, by using pseudo-redshifts, i.e. redshifts approximated from relations such as the lag-luminosity correlation. We select 630 Swift bursts (204 with known redshift, and 131 with both redshift and X-ray flux), collated from Nysewander et al. 2007, Butler et al. 2010, and the Swift burst archive. The redshift distribution is shown in Figure 6.
4.1 Flux Cut

Due to the restricted sensitivity of GRB detectors, our data is flux-limited, meaning that we cannot detect bursts dimmer than a certain threshold. In other words, our data is only representative of populations \( f > f_{\text{min}} \) or \( L > 4\pi d_L^2(z, \Omega_i)f_{\text{min}} \). As described above, the effects of the truncation can be corrected for using the Efron & Petrosian methods. As discussed in Section 2, we must first determine the luminosity evolution and remove it from the data in order to proceed.
4.1.1 Results Using the Prompt Gamma-Ray Threshold Only

For our 204 Swift bursts with known redshift, we apply a gamma-ray peak flux cut of 0.3 ph/s, shown by the blue curve in Figure 7. This value was chosen so as to be as conservative as possible without cutting out too many sources from the sample.

With this truncated data, we use the Efron-Petrosian 1992 method to calculate the degree of correlation between luminosity and redshift. We can determine this by first choosing a functional form for the luminosity evolution, which is often a simple power-law dependence $g(z) = (1 + z)^\delta$. We can then make the transformation $L \to L_0 = L/g(z)$ and vary $\delta$ until the test statistic $\tau$ approaches 0, indicating that $L_0$ and $z$ are independent. Once $L$
Figure 7: A flux cut of 0.3 ph/cm2/s applied to our sample. The pink line shows the correlation obtained from a naïve line fit and the effects of the truncation boundary.

and $z$ are independent, we can use the Lynden-Bell $C^-$ method to correct for the missing populations.

### 4.1.2 Results Using the Combined Gamma- and X-Ray Threshold

To more accurately correct for the missing population, we can combine our observed gamma-ray luminosity limit with a similar threshold for the X-ray luminosity. In short, we apply a flux cut to the X-ray data and keep only the data that makes it through both cuts. Here the maximum redshift, which is determined by the luminosity equation, $L_i = 4\pi d_L^2(z_{\text{max},i}, \Omega_f) f_{\text{lim},i}$, is chosen from whichever band makes it smallest.
4.2 K-Correction

Before any further analysis, we must apply a K-correction to determine the luminosities of the Swift bursts, which are observed in the 15-150keV energy band, in their rest frames. Factoring in this K-correction, our luminosity relation becomes \( L = 4\pi d_L^2(z, \Omega_i) f/K \). The value of \( K \) is determined by the ratio

\[
K(z) = \frac{\int_{15 \text{ keV}}^{150 \text{ keV}} f(E) \, dE}{\int_{15 \text{ keV}/(1+z)}^{150 \text{ keV}/(1+z)} f(E) \, dE}
\]

Depending on the spectra of the specific burst, \( f(E) \) is fitted to either a power law, a power law with an exponential cutoff, or the Band model

**Power Law:** \( f(E) = A \left( \frac{E}{100} \right)^\alpha \)

**Power Law Exponential:** \( f(E) = A \left( \frac{E}{100} \right)^\alpha e^{-\frac{(2+\alpha)E}{E_{\text{peak}}}} \)

**Band Model:**

\[
\begin{align*}
A \left( \frac{E}{100} \right)^\alpha e^{-\frac{(2+\alpha)E}{E_{\text{peak}}}} & \quad E < E_{\text{break}} \\
A \left( \frac{E}{100} \right)^\beta e^{-(\frac{(\alpha-\beta)E_{\text{peak}}}{(2+\alpha)100} - \beta)} & \quad E \geq E_{\text{break}}
\end{align*}
\]

We use spectral parameters from Butler et al. (2010), to obtain the necessary parameters and perform the K-correction. As the plot below shows, the K-correction is not trivial in this case, particularly for high redshift.
5 Results

5.1 Luminosity Evolution

Figure 7 depicts the gamma-ray peak luminosity distribution, which shows a strong correlation $L(z) \propto (1 + z)^{3.9}$ (red line). This correlation is mostly due to the truncation induced by the flux limit (blue curve).

Using a conservative flux cut of $0.3 \text{ ph cm}^{-2} \text{s}^{-1}$, we apply the test of independence outlined in Section 2 to the full sample of Swift GRBs with known $z$ to derive the true luminosity evolution. A common functional form is the simple power law relation $g(z) = (1 + z)^\delta$. However, in recent works dealing with the luminosity evolution of quasars (Singal, et al. 2013), it was found that the evolution slows down at high redshift and $g(z) = L_0(1 + \ldots$
\(z^\delta/[1 + (1 + z)/(1 + z_{\text{crit}})^{0.66}]\) (which still preserves the single free parameter) is a better form. We use this form as well, with \(z_{\text{crit}} = 2.5\), and find the optimal value for the exponent to be \(\delta = 2.2\). Note that since \(g(0) = 1\), the luminosities \(L_0\) can be considered the de-evolved (or “local”) luminosities of the source at \(z = 0\).

### 5.2 Luminosity Function

Having established the independence of the modified luminosity \(L_0\) with \(z\), we now apply the Lynden Bell \(C^-\) method to obtain the “local” cumulative luminosity function \(\phi(L_0)\). The result is shown in Figure 9.

![Figure 9: Cumulative luminosity function \(\phi(L)\) vs. \(L\) (with arbitrary vertical normalization)](image)

Next, we differentiate the cumulative luminosity function to obtain the
differential luminosity function \( \psi(L_0) = -d\phi/dL_0 \), which represents the number of GRBs with luminosity between \( L \) and \( L + dL \) after accounting for the selection effect due to the flux bias. This is shown in Figure 10.

Figure 10: Luminosity function \( \psi(L) = -d\phi/dL \), representing the total number of bursts between \( L \) and \( L + dL \).

5.3 Co-Moving Density Rate Evolution

Similarly, we find the corrected cumulative redshift distribution \( \dot{\sigma}(z) \).

Next, we differentiate the cumulative density distribution \( \dot{\sigma}(z) \) to get \( \dot{\rho}(z) \).
Using the definition of $V(z)$ with the assumed cosmological parameters,

$$V(z) = \frac{4\pi}{3} \left[ \frac{c}{H_0} \int_0^z \sqrt{\frac{1}{\Omega_m(1+z')^3 + \Omega_\Lambda}} dz' \right]^3$$

we can derive the co-moving density rate evolution $\dot{\rho}(z)$ from the relation

$$\dot{\rho}(z) = \frac{d\sigma(z)}{dz}(1+z) \left( \frac{dV}{dz} \right)^{-1}$$

Various evaluations of the co-moving density rate evolution are shown in Figure 12. The solid red line is what we obtain if we ignore the effects of truncation, i.e. directly from the observed redshift distribution as
\[ \dot{\rho}(z) = (dN/dz)(1 + z)(dV/dZ)^{-1}. \]

The solid black line shows the rate evolution corrected for the bias introduced by the gamma-ray threshold alone, while the green line includes the effects of the X-ray afterglow detection threshold, as described in Section 4.1.2. We also show the rate evolution without the luminosity evolution correction (dashed blue line), demonstrating the significance of the L-z correlation, which is often overlooked.

Figure 12: Comparison of the co-moving density rate evolution $\dot{\rho}(z)$ for (1) luminosity evolution corrected and gamma-ray flux threshold used, (2) ignoring luminosity evolution, (3) luminosity evolution corrected and combined gamma- and X-ray thresholds used, and (4) raw smoothed distribution.

5.4 Comparison to SFR

The green curve, which is corrected for almost all biases, is compared with the SFR taken from Reddy, et. al. (2007) in Figure 13. As the figure
shows, there is considerably divergence between the GRB rate evolution and SFR for redshift below 3 when normalized at higher $z$ (lower curve), and at both high and low redshift when normalized at the SFR peak (upper curve).

Figure 13: $\dot{\rho}(z)$ compared to SFR data taken from Reddy, et al. (2007). The blue and red open circles are determinations of SFR($z$) based on rest-frame UV and far-infrared, respectively. The vertical scaling is arbitrary, so $\dot{\rho}(z)$ is shown normalized to SFR at high $z$ as well as at the SFR peak.

### 5.5 logN-logS and Comparison to BATSE

In Figure 14, we show the logN-logS histogram (i.e., the cumulative number of sources detectable at a given flux sensitivity) of our 204 Swift bursts with known redshift (red diamonds), over which our predicted source counts (black line) are superimposed. This prediction is based on the luminosity
function and co-moving density rate evolution, derived earlier in the paper, as 
\[ N(> f) = \int_0^\infty \frac{dV}{dz} \phi(L_{\text{min}}(z, f)) \, dz, \]
where \( L_{\text{min}}(z, f) = 4\pi d_L^2 f_{min}/g(z) \).

Its agreement with the observed source counts provides a nice test of the self-consistency of our results. However, these distributions do not agree with the logN-logS histogram of our larger sample of 630 Swift GRBs (with well defined flux limits but not all having known redshift), nor with the sample of 2,135 BATSE bursts, though they agree with each other surprisingly well. The cause of this disagreement and its relation to the disagreement between the co-moving density rate evolution and SFR must be investigated further.

Figure 14: logN-logS plot showing agreement between observed vs. predicted source counts for our 204 Swift bursts with known redshift. We also compare this to the observed source counts for a larger 630 burst Swift sample and a 2135 burst BATSE sample, which agree with each other but not our sample.
6 Discussion

The results presented in Section 5 raise some interesting and important questions about the cosmological evolution of GRBs which we intend to explore in future works.

Firstly, the cause of the disagreement between the GRB co-moving density rate evolution and the SFR should be investigated further, as it is a new and important result. The persistent discrepancy at redshift \( < 2 \) regardless of normalization seems to suggest a distinct class of low-z GRBs, but our luminosity function shows no signs of high or low luminosity bimodality, appearing to be fit by a smooth broken power-law, and tests carried out in Petrosian, et al. 2009 offer evidence against this as well. There is also the possibility of bimodality in the redshift distribution, which is not ruled out by the data. In Figure 15 below, we compare the observed redshift distribution of our 204 Swift bursts to the corrected distributions using both gamma-ray-only and combined gamma- and X-ray detection thresholds. The uncorrected distribution reveals a deficiency of GRBs in the \( 1.5 < z < 2 \) band, a region sometimes referred to in literature as the “redshift desert”. This is reflected also in the flattening of the \( \sigma(z) \) curves in that area. However, whether this feature reflects a bimodality between redshifts below and above \( z \sim 2 \) or merely the absence of easily identifiable spectral lines in that range also needs to be investigated further.

Another discrepancy which must be explored is the \( \log N \)-\( \log S \) result.
Figure 15: $d|\sigma/dz$ from raw distribution compared to corrected distributions using gamma-ray threshold as well as combined gamma- and x-ray thresholds. The cause of the deficiency in the band $1.5 < z < 2$ must be investigated further.

While our prediction agrees nicely with the observed source counts for our 204 Swift bursts with known redshift, confirming self-consistency, it does not agree with the logN-logS curves for all Swift or BATSE GRB samples, which seem to agree with each other better than expected (see also Dai 2009). The source of this discrepancy and its relation to the SFR discrepancy must be examined.

In addition to the bias involved in the detection of the prompt emission, the measurement of the redshift can introduce further truncations. X-ray and optical afterglows are vital for timely and accurate burst localization and redshift measurement. In this paper, we used the X-ray data to help
us estimate these additional selection biases, but a more rigorous treatment would require us to obtain the evolution of the bivariate luminosity function \( \psi(L_\gamma, L_x, z) \), with multiple flux limits rather than the simpler case we were considering. For the analysis to be complete, we must determine the correlation between luminosities as well as the luminosity evolution for each band. Nevertheless, the Efron-Petrosian method illustrated above can be generalized to higher dimensions without much difficulty. We must decorrelate the luminosities \( L_\gamma, L_x \) using the same method that we used to transform the monovariate luminosity \( L \) from \( z \). Thus we will obtain the monovariate luminosity functions, which will in turn give us the multivariate luminosity function. We propose to apply this in a future work. Once we better understand the optical flux thresholds, which are not as well-defined as the gamma-ray flux thresholds but seem to be strongly correlated with the better known X-ray flux thresholds, we can include optical luminosities as well.

7 Summary

To use GRBs as cosmological probes, we need a strong understanding of the distribution and evolution of their basic characteristics. In this work, we have emphasized the advantages of a non-parametric approach for determining the luminosity and rate density evolutions. Using these techniques, we found a co-moving rate density evolution that is startlingly different from the evolution of the star formation rate. Further studies are required to improve
our understanding of this phenomenon, which will help in using GRBs as tools to explore the high redshift universe.

8 References


• Jakobsson, P. et al. 2006. A mean redshift of 2.8 for Swift gamma-ray bursts. Astronomy & Astrophysics, 447, 897


• Lloyd, N. M., Petrosian, V. & Mallozzi, R.S. 2000. Cosmological versus intrinsic: the correlation between intensity and the peak of the $\nu F_\nu$ spectrum of

• Lloyd, N. M., Fryer, C. L. & Ramirez-Ruiz, E. 2002. Cosmological aspects of gamma-ray bursts: luminosity evolution and an estimate of the star formation rate at


• Malmquist, K. G. 1920, Medd. Lund. Obs., Ser. 2, No. 22


